$\therefore$ Side AB=side AC, being sides opposite equal angles, and therefore $\triangle\ ABC$ is isosceles. Q. E. D.

Also solved by *J. A. Calderhead, J. R. Baldwin, Josiah H. Drummond, H. M. Cash, J. F W. Scheffer*, and *G. B. M. Zerr*.

PROBLEMS.

28. **Proposed by Professor HENRY HEATON, M. S., Atlantic, Iowa.**

Through three given points to pass two spherical surfaces tangent to a giver sphere.

29. **Proposed by H. W. HOLYCROSS, Superintendent of Schools, Pottersburg, Ohio.**

If the two angles at the base of a triangle are bisected; and through the point of meeting of the bisectors a line is drawn parallel to the base, the length of the parallel between the sides is equal to the sum of the segments of the sides between the parallel and the base.

30. **Proposed by CHARLES E. MYERS, Canton, Ohio.**

A circle containing one acre is cut by another whose center is on the circumference of the given circle, and the area common to both is one-half acre. Find the radius of the cutting circle.

CALCULUS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS TO PROBLEMS.

7. **Proposed by Professor J. F. W. SCHEFFER, A. M., Hagerstown, Maryland.**

To determine the function $F(x)$ so that $F(x+y)\times F(x-y)=[F(x)]^2-[F(y)]^2$.

Solution by the Proposer.

Putting $x+y=z$, $x-y=t$, we get

$F(z).\ F(t)=[F(x)]^2-[F(y)]^2$. Differentiating this equation twice according to z and t as independent variables, and considering that $\frac{dx}{dz}=\frac{1}{2},\frac{dx}{dt}=\frac{1}{2},\frac{dy}{dz}=\frac{1}{2},\frac{dy}{dt}=-\frac{1}{2}$; we obtain $F''(z)F(t)F''(t).F(z)$.

$\therefore \frac{F''(z)}{F\ (z)}=$ constant $=a^2$. Denoting $F(z)$ or $F(x)$ by u, we have the differential equation $\frac{d^2u}{dx^2}=a^2u$. $\therefore\ u=Ce^{ax}+C'e^{-ax}$. Since $F(0)=0$, we have $C'=-C$, $\therefore\ u=C(e^{ax}-e^{-ax})=C\sin bx$, where C and b designate any two constant quantities.